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ABSTRACT

This paper discusses uses of TI-73 graphing calculators for middle school mathematics students. It indicates that with the appropriate use of the TI-73, students can develop understanding about variables and basic concepts of algebra, and explore mathematical topics. Background on middle school students' difficulties with those subjects is provided along with examples of teaching some middle school mathematics concepts with the TI-73 such as order of operations, solving equations, and variables. (ASK)

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Using Technology to Prepare All Students for Success in Algebra

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Introduction

The TI-73 is a middle school (and upper elementary in my opinion) graphing calculator. It has many of the important features of the Math Explorer Plus and the TI-80 graphing calculator. It also has many new data analysis features. It is a flash ROM based product which means the chip can be changed electronically, and applications can be installed electronic.

More importantly, with appropriate use of the TI-73, students can develop understanding about variables, basic concepts of algebra, and explore mathematical topics. The key is to introduce these ideas through numerical computation and problem solving involving a balance of paper-and-pencil and calculator techniques.

Background

Students have difficulty with algebra because of their lack of understanding of variables. Researchers generally agree that there are four levels of understanding about variable. For example, Level 1 means that a variable or letter is understood to stand for a specific object, or number. Lesley R. Booth gives a report of the strategies and errors in secondary mathematics in England in a monograph titled, Algebra: Children's Strategies and Errors (1984). The students in this project were 13 to 15 year olds. The list of references in this monograph provide a rich source of information about children misunderstanding about algebra.

Here is an example of a student at Level 1 in understanding about variables reported by Booth. The Interviewer is denoted by I, and the 15-year-old student, Trevor, by T. The question posed was, "Can $x + y + z$ ever equal $x + p + z$?"

T: 'It won't be true, never.'
I: 'Never?'
T: 'Never, because, it'll have different values… because p has to have a different value from y and the other values, so that'll never be true.'
I: 'So p has to have a different value… why do you say that?'
T: 'Well, if it didn't have a different value, then you wouldn't put p, you'd put y. You see, you put a different letter for every different value.'

Of the 35 students interviewed, 23 (about 66%) had the same understanding about variable. That is, a variable stands for a specific number.

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Another interesting question asked was. "What is the perimeter of the shape with n sides all of length 2?" This time the student is 14-year-old Michelle (M).

I: 'Can you write down anything for the perimeter?'
M: 'No, because n probably stands for a number and if that's n, I can't get how many sides that is. And I've got to add up all the sides, all the 2's, so I need to know how many. Unless n stands for … like say n's in the alphabet, somewhere along the numbers … if n stands for one of those, then you can say what n is.'
I: 'How do you mean?'
M: 'Well, say n's 14?'
I: 'How did you get that?'
M: 'Same as I said before, to get n for a number, I got 14. It's 14 along.'
I: 'Oh, did you count along the alphabet?'
M: 'Yes, so that's 14, so I took that for n, so it should be … 28, to make up the quantity, the perimeter.'
I: '28?'
M: 'Yes, you add up all the 2's.'
I: Could n be another number, or does it *have* to be 14?'
M: 'Well it could be any number, but if you write the question out like that, without any indication of the number, then there's nothing The only thing you can do is go along the alphabet and take whatever number n comes under to tell you how many two's you need.'

Only about 11% of the students got this type of item correct.

Order of Operations

Using a calculator with algebraic logic with hierarchy requires students to understand basic arithmetic properties that are important in algebra. We have all had students who report the incorrect answer 36 to the following problem:

Evaluate $2x^2$ for $x = 3$.

The student performs the operations in order from left to right to get the incorrect answer. That is, multiple 2 by 3 to get 6, and then square 6 to get 36. Appropriate use of technology can help students gain a deeper understand about order of operations.

Example 1 Show that addition and multiplication cannot be interchanged. In other words, multiplication has higher hierarchy than addition.

Solution Set the constants C1 and C2 as shown in Figure 1a. The first line of Figure 1b shows the result of entering 3 and applying C1 followed by C2. The second line shows the reverse order. By experimenting, students can discover that the order in which operations are applied matters.

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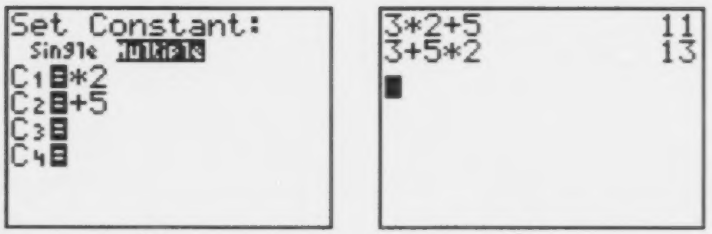

(a) (b)

Figure 1 (a) Setting multiple constants. (b) Pressing 3, CONST, 1:C1, and 2:C2 gives 11. Reversing the last two keys gives 13.

Students can do numerous activities like Example 1 to truly understanding the meaning of the oft given teacher reminder, remember "My Dear Aunt Sally," when students are doing paper-and-pencil computation. Students are actually able to discover the rules and hierarchy themselves.

Solving Equations

We found in earlier projects that using guess-and-check or guess-and-test was an effective technology based approach to help students understand the meaning of solving equations. The Constant feature of the TI-73 is very useful for this method.

Example 2 Solve $2x + 5 = 14.5$ using the Constant feature of the TI-73.

Solution Enter $\times 2 + 5$ for the single constant C1. Figure 2 shows the result of using guess-and-check to solve the equation.

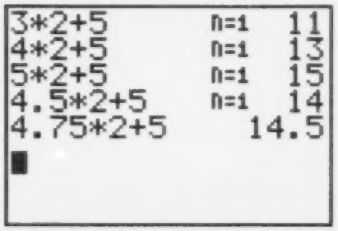

Figure 2 The result of pressing 3, CONST, then 4, CONST, and so forth.

It is important to start with problems that have answers that can be obtained quickly. Then, students can be given more difficult problems. This method helps students build number sense.

Using Lists to Introduce Variables

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Using LIST and STAT is a great way to introduce graphing with students. I will not go into this in this paper.

List based arithmetic can help students deepen their levels of understanding about variables. Consider a rectangle whose length is 5 more than its width. In our earlier work in the Approaching Algebra Numerically (AAN) Project (Demana, Leitzel, & Osborne, 1988) with seventh and eighth graders, we had students build tables like the following one that shows some of the possible values for perimeter and area. We found that the project students had little understanding about area and perimeter, and frequently interchanged them. That is why we usually made tables with columns for both area and perimeter.

Width	Length	Perimeter	Area
5	5 + 5 = 10	5 + 10 + 5 + 10 = 30	50
10	10 + 5 = 15	2(10) + 2(15) = 50	150
15	15 + 5 = 20	2(15 + 20) = 70	300
20	20 + 5 = 25	4(20) + 10 = 90	500
w	$w + 5$		

We had a good deal of trouble getting students to write expressions such as 5 + 5, 10 + 5, and so forth in the second column. This is important if students are going to be able to write $w + 5$ in the last row. The study in England reported on earlier found that students would simplify $w + 5$ to $5w$. Objects like 5 + 10 and $w + 5$ were not legitimate expressions—rather they were directions to add. Notice the nice patterns in the second and third columns. This can be taken advantage of when solving problems.

In the third column, I showed some of the ways our project students would calculate the perimeter. Only two of my students used 4(20) + 10, and many used the "add them up" procedure in the second row. Several equivalent expressions result for perimeter in the last row.

$$P = w + (w + 5) + w + (w + 5)$$
$$P = 2w + 2(w + 5)$$
$$P = 2(w + (w + 5)$$
$$P = 4w + 10$$

When we solved problems about the perimeter, classroom discussion led to student understanding that these expressions were equivalent. They also found that the fourth one was "best" for solving problems about perimeter using guess-and-check. (The two students who initially used this method would always get the answers first.)

Unfortunately, graphing calculators were not yet invented when we were in this project. I would like to illustrate how we could have tackled this problem with the list feature of graphing calculators. Figure 3 shows the first three columns. The maximum number of letters possible for a column heading is 5, so I used L length and P for perimeter.

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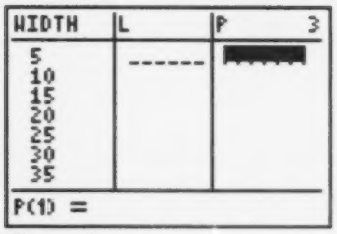
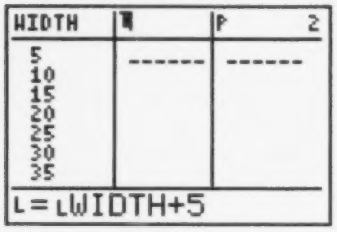
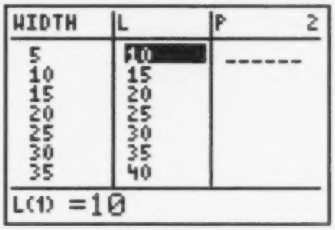


Figure 3

List based arithmetic allows us to use the fact that list L can be obtained from list WIDTH as shown in Figure 4a. Pressing ENTER results in all computations being done with the calculator.



(a)



(b)

Figure 4

Now students can complete the third column (P) using their own expression for perimeter (Figure 5). Comparing their P list with other students using different expressions helps convince them that the expressions for perimeter are equivalent.

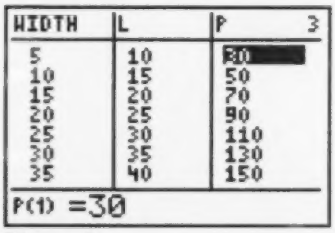


Figure 5

Let's look at the problem of finding one of these rectangle with perimeter 73. Students learn that solving such equations is just finding the right w so that 73 shows up in the P column. They can observe that the required w is between 15 and 20. This concept of "bracketing" is fundamental in mathematics of all levels. This approach allows students to experience this technique at an earlier age.

If students insert their guess, say 17, in the first column, they will find out that nothing happens to the second and third column. They would be forced to recalculate the second and third columns. Fortunately, there is an easy way to make the second column compute

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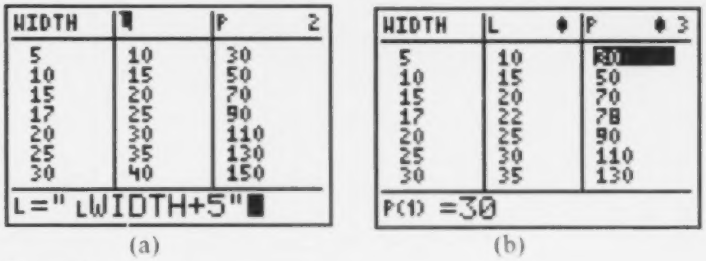
automatically when the first column is changed. We just need to enter the expression we used before with quotes around it as suggested in Figure 6a. Now any changes made in the first column will automatically be calculated in column 2. We do the same for the P column (Figure 6b).



(a) (b)

Figure 6

Now students can see that a width of 17 results in a perimeter of 78, a little two big. They can try 16, then 15.5, and their next guess might be 15.75 (Figure 7). Of course, students will proceed at their own rate and with their own guess. Eventually, the problem will be solved. That is, a rectangle with width 15.75, length 20.75 has a perimeter of 73.

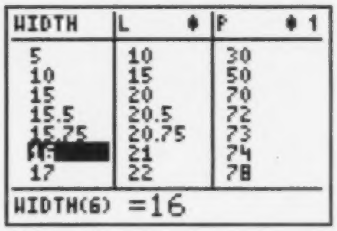


Figure 7

Besides solving this problem numerically, students will begin to appreciate that variables can be replaced by sets of values. Notice that this procedure illustrates that variables can be introduced using words like width, length, and perimeter.

Our results from the ANN project suggested that basic concepts of algebra are accessible to students through numerical computation and problem solving. In particular, the ideas of variable and function can be established to lay the foundation for success in algebra and advanced mathematics courses. Now with the power of graphing calculators, the successful approach we used can be enriched with activities similar to the ones illustrated in this paper.

Using Real Data

The use of real data can help students learn to value and appreciate mathematics. Use a variety of regression models to fit the data in the table below. Use the models to predict the population of the U.S. in 1990 and compare with the actual value of approximately 250 million.

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Year	Population (millions)
1880	50.2
1890	63.0
1900	76.0
1910	92.0
1920	105.7
1930	122.8
1940	131.7
1950	151.3
1960	179.3
1970	203.3

Source: The Statesman's Yearbook, 129th ed. (London: The Macmillian Press, Ltd, 1992).

Bibliography

Booth, Lesley R. *Algebra: Children's Strategies and Errors*. New Windsor, Berkshire, England: NFER-Nelson Publishing Co., 1984.

Collis, Kevin F. "Operational Thinking in Elementary Mathematics." In *Cognitive Development*, edited by John A. Keats, Kevin F. Collis, and Graeme S. Halford. New York: John Wiley & Sons, 1978.

Comstock, Margaret, and Franklin Demana. "The Calculator Is a Problem-solving Concept Developer." *Arithmetic Teacher*, Vol 34, No 6 (February 1987): 48-51.

Demana, Franklin D., and Joan R. Leitzel. "Establishing Fundamental Concepts through Numerical Problem Solving." In *The Ideas of Algebra*, 1988 Yearbook of the National Council of Teachers of Mathematics, pp. 61-68, Reston, Va.: The Council, 1988.

Demana, Franklin D., Joan R. Leitzel, and Alan Osborne, *Seventh and Eighth Grade Units—Approaching Algebra Numerically Project*. Student Materials and Teacher Materials. Lexington, Mass.:D. C. Heath & Co. (now Houghton Mifflin), 1988.

Demana, Franklin D., and Bert K. Waits. "Instructional Strategies and Delivery Systems." In *Algebra for Everyone*, edited by Edgar L. Edwards, Jr., The National Council of Teachers of Mathematics, pp. 53-61, Reston, Va.: The Council, 1990.

Hart, Kathleen. *Secondary School Children's Understanding of Mathematics*. London: Chelsea College, Centre for Science and Mathematics Education Research Monographs, 1980.

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Hart, K., and Lesley R. Booth. "Children Find Mathematics Difficult: The Results of the CSMS Research." *Bulletin of the Institute of Mathematics and Its Applications* 17 (May 1981): 114-15.

Küchemann , Dietmar E. "Children's Understanding of Numerical Variables." *Mathematics in School* 7 (September 1978): 23-26.

Leitzel, Joan R., and Alan Osborne. "Mathematical Alternatives for College Preparatory Students." In *The Secondary School Mathematics Curriculum*, 1985 Yearbook of the National Council of Teachers of Mathematics, pp. 150-65, Reston, Va.: The Council, 1985.

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